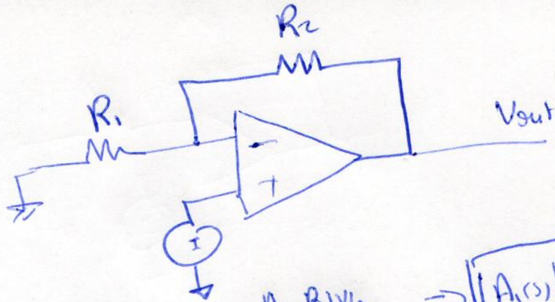
Problem 1

a)



* Open Loop $A_1(s) = \frac{GB}{s+BW_A} = \frac{A_0 BW_A}{s+BW_A} \Rightarrow \boxed{|A_1(s)| = \frac{A_0 BW_A}{\sqrt{\omega^2 + BW_A^2}}}$

At $\omega = 0$ $\boxed{A_1(s) = A_0}$

* Closed Loop $V_{out} = (V^+ - V^-) A_1(s) = \left(V_{in} - \frac{R_1 V_{out}}{R_1+R_2}\right)\frac{A_0 BW_A}{s+BW_A}$

$$V_{out}\left(1 + \frac{A_0 BW_A}{K_0(s+BW_A)}\right) = \frac{A_0 BW_A}{s+BW_A} V_{in}$$

$$A_{FB}(s) = \frac{A_0 BW_A/(s+BW_A)}{(K_0(s+BW_A)+A_0 BW_A)/K_0(s+BW_A)} = \frac{K_0 A_0 BW_A}{K_0(s+BW_A)+A_0 BW_A}$$

$$|A_{FB}(s)| = \left|\frac{A_0 BW_A}{s + BW_A + \frac{A_0 BW_A}{K_0}}\right| = \frac{A_0 BW_A}{\sqrt{\omega^2 + \left(BW_A + \frac{A_0 BW_A}{K_0}\right)^2}}$$

At $\omega = 0$ $A_{FB}(s) = \frac{A_0 BW_A}{BW_A\left(1 + \frac{A_0}{K_0}\right)} = \frac{A_0}{\frac{K_0 + A_0}{K_0}} = \frac{A_0 K_0}{A_0 + K_0} \approx \boxed{K_0}$

$$\frac{|A_1(s)|}{|A_{FB}(s)|} = \frac{\sqrt{\omega^2 + \left(BW_A + \frac{A_0 BW_A}{K_0}\right)^2}}{\sqrt{\omega^2 + BW_A^2}} = \frac{\sqrt{\omega^2 + \left(BW_A + \frac{GB}{K_0}\right)^2}}{\sqrt{\omega^2 + BW_A^2}}$$

~~$GB = BW \cdot K_0 \Rightarrow \frac{GB}{K_0} = BW$~~

At $\omega \gg BW$ $\begin{cases} \omega^2 + \left(BW_A + \frac{GB}{K_0}\right)^2 \approx \omega^2 \\ \omega^2 + BW_A^2 \approx \omega^2 \end{cases}$

$\Rightarrow \boxed{\frac{|A_1(s)|}{|A_{FB}(s)|} = 1}$ $\underline{\omega \gg BW}$

b) $V_{out} = (V^+ - V^-)A(s) = \left(V_{in} - \frac{R_1 V_{out}}{R_1 + R_2}\right)\frac{GB}{s}$

$$V_{out}\left(1 + \frac{GB}{K_o s}\right) = \frac{GB}{s} V_{in}$$

$$A(s) = \frac{V_{out}}{V_{in}} = \frac{GB/s}{\left(\frac{K_o s + GB}{K_o s}\right)} = \frac{K_o\, GB}{K_o s + GB} = \frac{GB}{s + \left(\frac{GB}{K_o}\right)}$$

$$\boxed{|A(s)| = \frac{GB}{\sqrt{\omega^2 + \left(\frac{GB}{K_o}\right)^2}}}$$

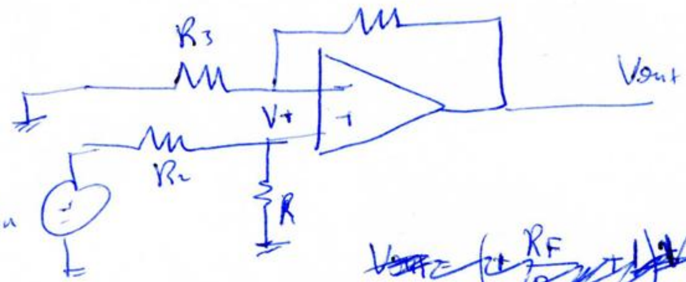
Compare to $A_{FB}(s) = \frac{GB}{\sqrt{\omega^2 + \left(BW_A + \frac{GB}{K_o}\right)^2}}$

~~BW_A~~ ~~...~~ ω_{op}

At frequencies $\omega \gg BW_A$ $\quad |A(s)| \simeq A_{FB}(s)$

Problem 2

a)



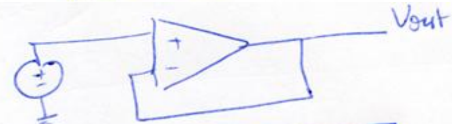
~~$V_{out} = \left(1 + \frac{R_F}{R_3} \ldots\right) V_{in} =$~~

$$V_{out} = \left(1 + \frac{R_F}{R_3}\right) V^+ = 2V^+$$

$$\boxed{K_o = \frac{V_{out}}{V^+} = 2}$$

$$\boxed{BW = \frac{GB}{K_o} = \frac{130\,KHZ}{2} = 65\,KHZ}$$

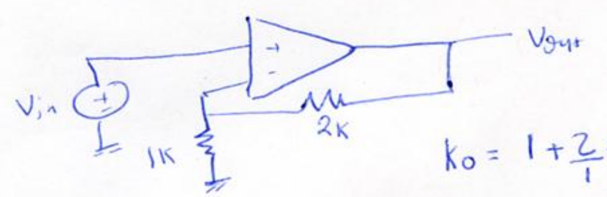
b)



$\Rightarrow K_o = 1$

$$\boxed{BW = GB = 130\,KHZ}$$

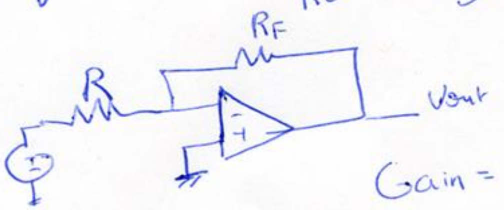
c)



$K_o = 1 + \frac{2}{1} = 3$

Non-inverting $\Rightarrow BW = \frac{GB}{K_o} = \frac{130}{3} = \boxed{43\,KHZ}$

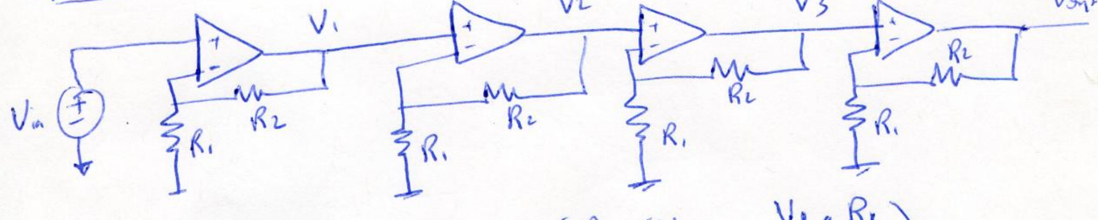
d)



$\text{Gain} = \frac{60}{20} = 3$

Inverting $\quad BW = \frac{GB}{1+K_o} = \frac{130}{4} = \boxed{32.5\,KHZ}$

Problem 3



$$V_1 = \frac{GB}{s}\left(V^+ - V^-\right) = \frac{GB}{s}\left(V_{in} - \frac{V_1 \cdot R_1}{R_1+R_2}\right)$$

$$V_1\left(1 + \frac{GB\,R_1}{s(R_1+R_2)}\right) = \frac{GB}{s} V_{in} \Rightarrow V_1 = \frac{GB/s}{\left(\frac{s(R_1+R_2)+GB\,R_1}{s(R_1+R_2)}\right)} V_{in}$$

or $$\boxed{V_1 = \frac{(R_1+R_2)\,GB}{s(R_1+R_2)+GB\,R_1} = \frac{GB}{s + GB\left(\frac{R_1}{R_1+R_2}\right)} V_{in}}$$

Similarely $$\boxed{V_2 = \frac{GB}{s + GB\left(\frac{R_1}{R_1+R_2}\right)} V_1}$$

" $$\boxed{V_3 = \frac{GB}{s + GB\left(\frac{R_1}{R_1+R_2}\right)} V_2}$$

$$V_{out} = \frac{GB}{s + GB\left(\frac{R_1}{R_1+R_2}\right)} V_3 = \frac{GB^4}{\left[s + GB\left(\frac{R_1}{R_1+R_2}\right)\right]^4} V_{in} \Rightarrow \boxed{T(j\omega) = \frac{GB^4}{\left[j\omega + GB\left(\frac{R_1}{R_1+R_2}\right) \ldots\right.}}$$

$$\boxed{K_0 = \left|\frac{V_{out}}{V_{in}}\right|_{\omega=0} = \frac{1}{\left(\frac{R_1}{R_1+R_2}\right)^4} = \left(\frac{R_1+R_2}{R_1}\right)^4}$$

$$|T(j\omega)| = \frac{K_0}{\sqrt{2}} \Rightarrow \frac{GB^4}{\left(\omega_B^2 + \left(\frac{GB\,R_1}{R_1+R_2}\right)^2\right)^2} = \frac{\left(\frac{R_1+R_2}{R_1}\right)^4}{\sqrt{2}}$$

$$\Rightarrow \left(\frac{R_1+R_2}{R_1}\right)^2\left(\omega_B^2 + \left(\frac{GB\,R_1}{R_1+R_2}\right)^2\right) = 2^{1/4}\,GB^2$$

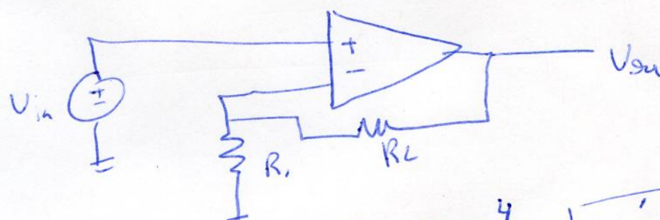
$$\Rightarrow \omega^2 = GB^2\left(2^{1/4} - \left(\frac{R_1}{R_1+R_2}\right)^2\right)$$

$$\omega_B^2 = \frac{2^{1/4}\, GB^2}{\left(\frac{R_1+R_2}{R_1}\right)^2} - GB^2\left(\frac{R_1}{R_1+R_2}\right)^2$$

$$\omega_B = \sqrt{GB^2\left(\left(\frac{R_1}{R_1+R_2}\right)^{2} 2^{1/4} - \left(\frac{R_1}{R_1+R_2}\right)^2\right)}$$

$$\omega_B = GB\left(\frac{R_1}{R_1+R_2}\right)\left(2^{1/4} - 1\right)$$

$$\boxed{\omega_B = 0.18\, GB\left(\frac{R_1}{R_1+R_2}\right) = BW} \quad (1)$$

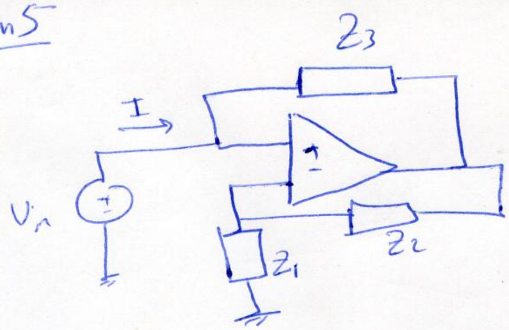


for a gain of $\left(\frac{R_1+R_2}{R_1}\right)^4$, $\boxed{BW' = \frac{GB}{\left(\frac{R_1+R_2}{R_1}\right)^4}} \quad (2)$

$$\boxed{\frac{BW}{BW'} = 0.18\left(\frac{R_1+R_2}{R_1}\right)^3}$$

Therefore the bandwidth is much larger when we cascade 4 stages.

Problem 5



$$\Rightarrow Z_{in} = -\frac{Z_1 Z_3}{Z_2}$$

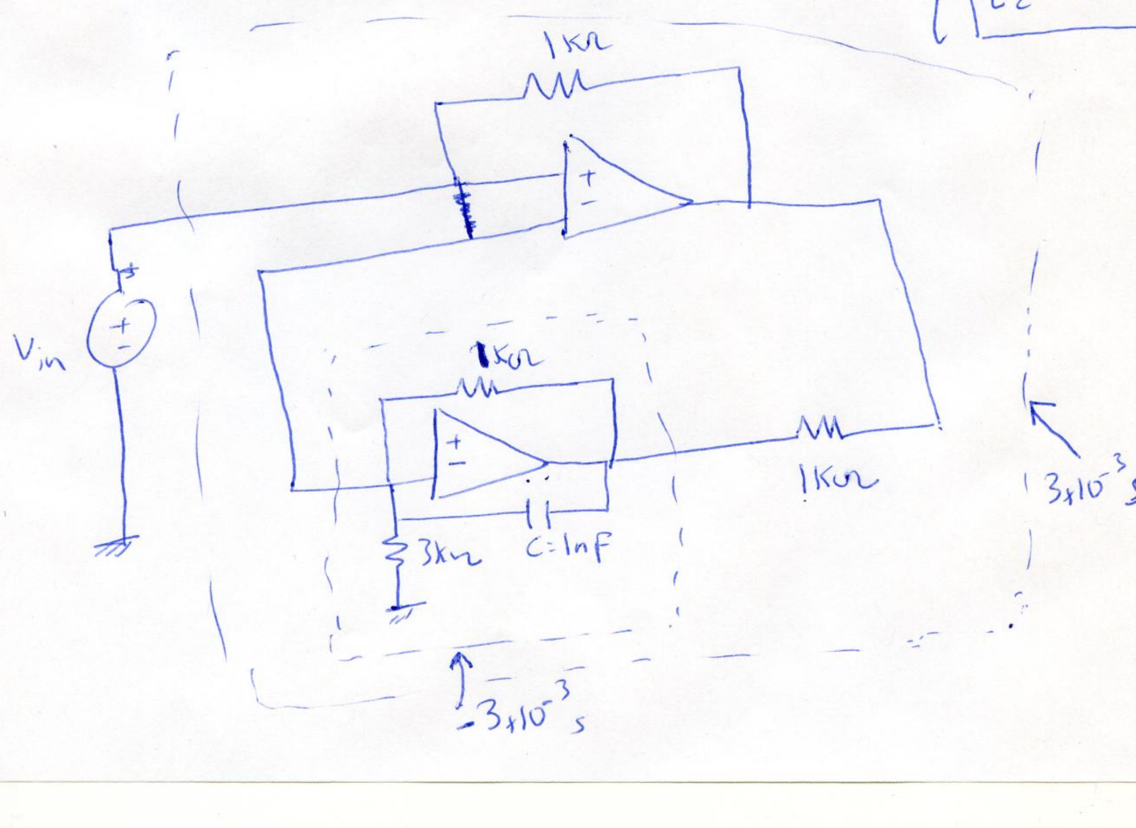
Choose $Z_2 = \frac{1}{sC}$, $Z_1 = R_1$, $Z_3 = R_3 \Rightarrow Z_{in} = -sCR_1R_2$

let $C = 1nF$ $R_1 = 3k\Omega$ $R_3 = 1k\Omega \Rightarrow Z'_{in} = -3\times10^{-3}s$

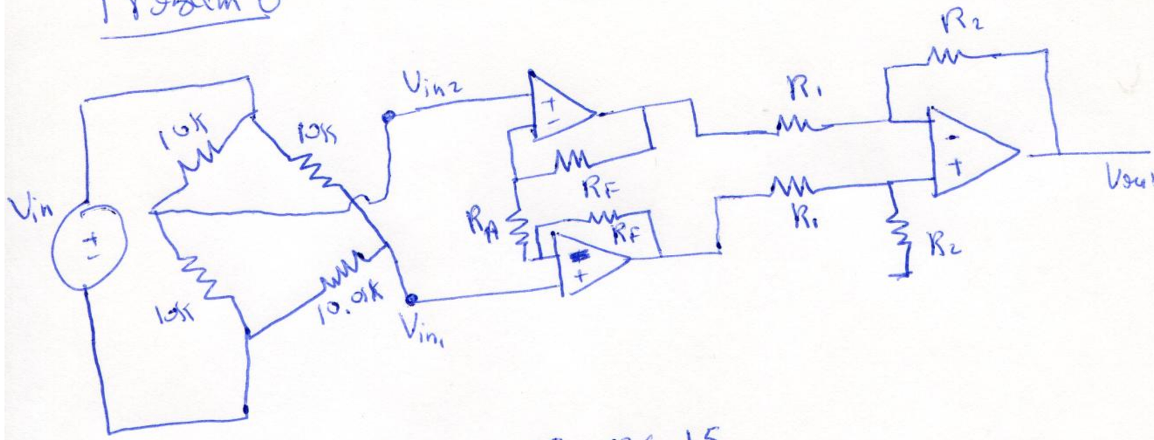
For a 3mH inductor $Z = 3\times10^{-3}s$

We need to invert Z'_{in}

$$Z_{in} = -\frac{Z'_{in} Z_3}{Z_2}$$ by letting $Z'_{in} = Z_1$, $Z_3 = 1k\Omega$, $Z_2 = 1k\Omega$



Problem 6



From Lecture 15 pg 15

$$V_{out} = (V_{in2} - V_{in1})\left(1 + \frac{2R_F}{R_A}\right)\left(\frac{R_2}{R_1}\right)$$

$$\boxed{V_{out} = (V_{in2} - V_{in1}) = 450(V_{in2} - V_{in1})}$$

$$V_{in2} = V_{in}\left(\frac{10K}{10K + 10K}\right) = \frac{1}{2} V_{in}$$

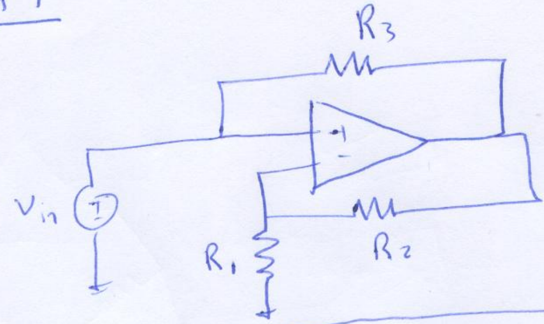
$$V_{in1} = V_{in}\left(\frac{10.01K}{10.01 + 10}\right) = 0.50025\, V_{in}$$

$$V_{out} = 450\, V_{in}(0.5 - 0.50025)$$

$$V_{out} = 0.1124\, V_{in}$$

$$\boxed{V_{out} = 0.56 \sin 500t}$$

Problem 7



$$R_{in} = -\frac{R_1 R_3}{R_2}$$

Choose $R_1 = 10\,k\Omega$ $R_3 = 1k\Omega$ $R_2 = 1k\Omega$

Problem 8 $\quad \varepsilon = \frac{\Delta L}{L} = \frac{6FL}{YWH^2} \qquad \frac{\Delta R}{R} = \frac{\varepsilon}{K}$

$L = 1\,m \qquad W = 1\,inch = 0.0254\,m \qquad H = 0.2\,inch = 0.00508\,m \qquad Y = 6.9 \times 10^{10} \frac{N}{m^2}$

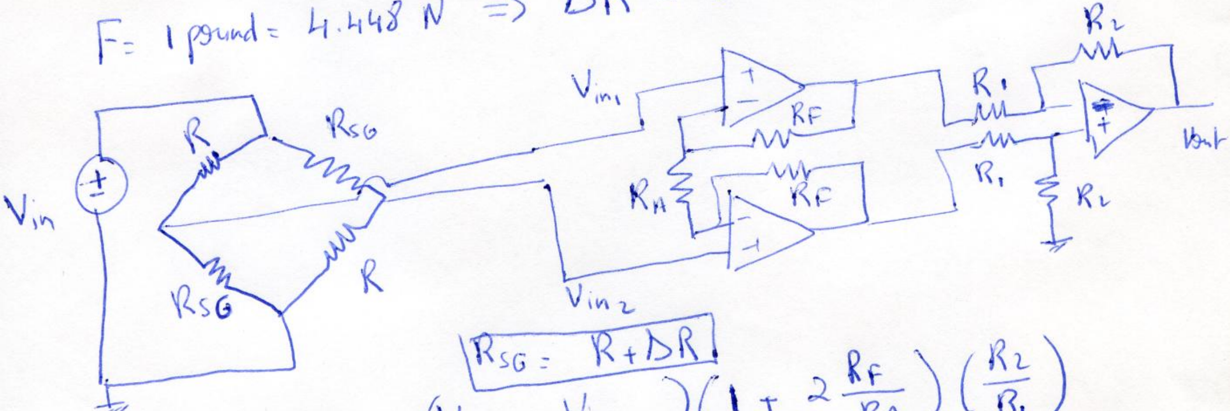
$1\ pound = 4.448\,N$

$$\varepsilon = F \frac{6 \times 1}{6.9 \times 10^{10} \times 0.0254 \times (0.00508)^2} = \frac{F\,6}{45228.3} = 1.327 \times 10^{-4}\,F$$

$$\frac{\Delta R}{R} = \frac{\varepsilon}{K} \Rightarrow \Delta R = \frac{R\,\varepsilon}{K} = \frac{300 \times 1.327 \times 10^{-4}}{1.5} \cdot F$$

$$\boxed{\Delta R = 0.0265\,F}$$

$F = 1\ pound = 4.448\,N \Rightarrow \Delta R = 0.117872\,\Omega$



$$\boxed{R_{SG} = R + \Delta R}$$

$$V_{out} = \left(V_{in_2} - V_{in_1}\right)\left(1 + 2\frac{R_F}{R_A}\right)\left(\frac{R_2}{R_1}\right)$$

$$V_{out} = \left(V_{in}\frac{R_{SG}}{R_{SG}+R} - V_{in}\frac{R}{R_{SG}+R}\right)\left(1 + 2\frac{R_F}{R_A}\right)\left(\frac{R_2}{R_1}\right)$$

$$V_{out} = \frac{V_{in}}{R_{SG}+R}\left(-R + R_{SG}\right)\left(1 + 2\frac{R_F}{R_A}\right)\left(\frac{R_2}{R_1}\right)$$

$V_{out} = 5V$ and $\Delta R = 0.117872\,\Omega \Rightarrow 5 = \frac{V_{in}}{600 + 0.117872}(0.117872)\left(1 + \frac{2R_F}{R_A}\right)\left(\frac{R_2}{R_1}\right)$

$$\Rightarrow 25456.33 = \left(1 + \frac{2R_F}{R_A}\right)\left(\frac{R_2}{R_1}\right) V_{in}$$

Choose $R_2 = 100\,K\Omega \quad R_1 = 1\,K\Omega \quad$ ~~$R_F = 250\,K\Omega$~~ ~~$R_A = 1\,K\Omega$~~

$V_{in} = 10\,V \quad$ ~~$R_F = 1.25\,K\Omega$~~ $\quad R_F = 1\,K\Omega \quad R_A = 1.3\,K\Omega$

Problem 9

$$\varepsilon = 8\times10^{-4} \Rightarrow \Delta R = \frac{\varepsilon R}{K} = \frac{8\times10^{-4}\times300}{1.5} = 0.16\,\Omega$$

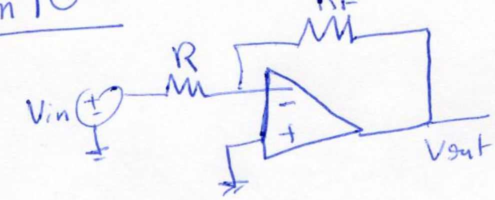
$$V_{out} = \frac{V_{in}\,\Delta R}{2R+\Delta R}\left(1+\frac{2R_F}{R_A}\right)\left(\frac{R_2}{R_1}\right)V_{in}$$

$$V_{out} = \frac{10\times0.16}{600.16}\left(1+\frac{2\times1}{1.3}\right)(100)\times10$$

$$\boxed{V_{out} = 6.77\ V}$$

Problem 10

a)

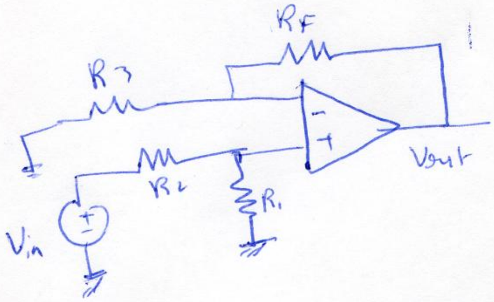


Short Vin ⇓

$V_{out\text{-}offset} = \left(1 + \frac{R_F}{R}\right) V_{os} = 4 \times 3\,mV$

$\boxed{V_{out\text{-}offset} = 12\,mV}$

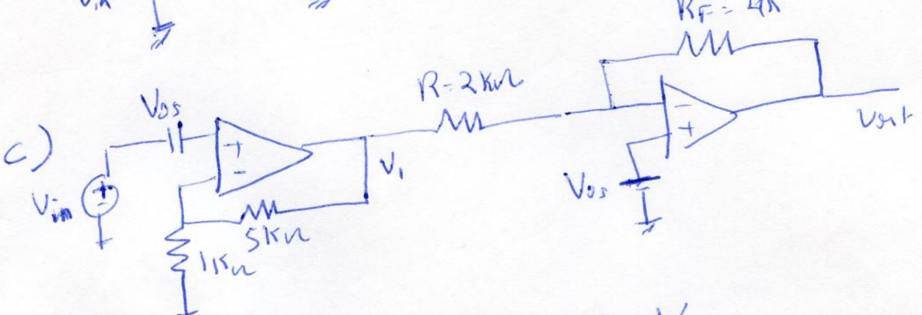
b)



Same as above after shorting Vin

$\boxed{V_{out\text{-}offset} = 12\,mV}$

c)



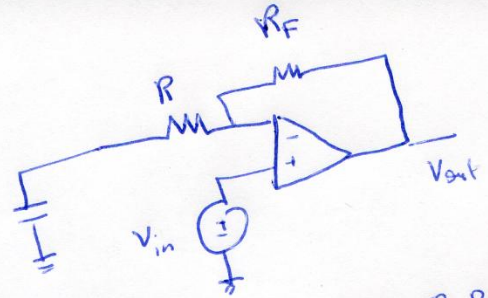
Short Vin ⇒ $V_1 = \left(1 + \frac{5}{1}\right) 3\,mV = 18\,mV$

in 2nd stage apply superposition ⇒ $V_{out\text{-}offset} = \left(-\frac{R_F}{R} V_1\right) + \left(1 + \frac{R_F}{R}\right) V_{os}$

$V_{out\text{-}offset} = -2 \times 18\,mV + 3 \times 3\,mV$

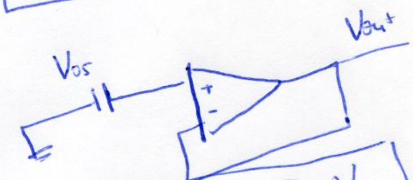
$\boxed{V_{out\text{-}offset} = -27\,mV}$

d)



from p13 Lecture 20: $V_{out\text{-}offset} = \frac{1 + sC(R + R_F)}{1 + sRC} V_{os}$

at $\omega = 0$ $\boxed{V_{out\text{-}offset} = V_{os} = 3mV}$



e)

$\boxed{V_{out} = V_{os} = 3mV}$